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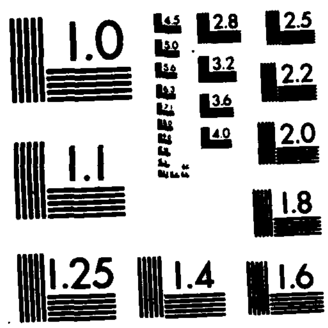

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On A Simple Adaptive Tracking Filter[1]

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Abstract

This paper presents a simplified adaptive tracking filter for a maneuvering target. The filter is very simple and easy to implement. No prior knowledge of the maneuvering characteristics is needed; the tracking errors are quite small when the target maneuvers. While the target does not maneuver, the tracking errors are a bit larger than others, but are still much smaller than measurement errors. This method can be used either with linear measurements or nonlinear measurements. Simulation results for various target maneuvers are presented.

1. Research sponsored by ONR Contract No. N00014-81-K-084.

2. R. R. Mohler, an IEEE Fellow, NAVELEX Professor of Electrical Engineering, Naval Postgraduate School, Monterey, CA 93943 from August 1983 to July 1984.

ON A SIMPLE ADAPTIVE TRACKING FILTER

I. INTRODUCTION

The problem of tracking a maneuvering target has been discussed by many authors [1] - [7]. For most of these filters, the maneuvering characteristics of the target are assumed. When the actual target maneuver does not meet the assumption, the tracking accuracy usually is poor.

Chan et al [6], [7] proposed a tracking scheme with input estimation, in which a model of the maneuvering characteristics is not needed. They estimate the acceleration (maneuver) inputs from the residuals and use a detector for checking the magnitude of the estimated input. Only when the magnitude exceeds the preset threshold level are the estimates used to update the Kalman filter. However, there are some weaknesses in this scheme. The first one is that when the target manuevers significantly, the rms errors in position are very large. The second is that when the target performs a small maneuver which lasts some time, the acceleration will be so small that the estimates will not exceed the threshold level and will not update the Kalman filter; after a long time the errors will get larger and larger. Lastly, the detector is somewhat complicated.

A simplified adaptive tracking filter is presented here which uses a simplified target model. It is assumed that the target moves in a straight line with constant speed. Also, smoothed residuals are utilized to control the gain calculation of the Kalman filter. Then, based on the characteristics of the feedback system, the filter decreases the tracking errors automatically. In this method, neither the characteristics of the maneuver nor the assumptions about the acceleration is required. The algorithm is very simple and easy to implement. When the target performs a maneuver, the rms value of position

errors are much smaller when compared with Chan's results [6]. When the target does not maneuver, the errors are a little larger than Chan's but still less than the measurement errors. This method also can be generalized to the case of nonlinear measurements by using the extended Kalman Filter or a "bilinear" type of filter [8], [9].

II. AN ADAPTIVE TRACKING FILTER

Consider the motion of a point mass in the plane with constant velocity as the target model, which can be described by the linear difference equations, i.e.,

$$X_{N+1} = FX_N + GW_N. \tag{1}$$

where $X_N' = [x(NT), \dot{x}(NT), y(NT), \dot{y}(NT)]$ with prime and dot designating transpose and time derivative, respectively; $x(t)$, $y(t)$ are the target Cartesian Coordinates; T is sampling time of measurements; W_N is a zero-mean Gaussian random vector with covariance $E\{W_N W_N'\} = \Gamma_N$ (in the experiments, it is assumed that $GW_N = 0$), and

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

At first, assume the measurements have been converted to rectangular coordinates. The measurement noise is an additive, zero-mean, white, Gaussian random vector, so that the measurement equations are

$$Z_N = HX_N + V_N \tag{3}$$

where

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{4}$$

$$E\{V_N\} = 0. \qquad E\{V_N V_K'\} = R_N \delta_{NK}, \tag{5}$$

where δ_{NK} is the Kronecker-delta function.

The estimates of the state at the time NT of the original Kalman filter are [10]

$$\hat{X}_N = F\hat{X}_{N-1} + K_N(Z_N - HF\hat{X}_{N-1}) = F\hat{X}_{N-1} + K_N S_N, \tag{6}$$

$$K_N = FP_{N-1}F'H'[HFP_{N-1}F'H' + R_N]^{-1}, \tag{7}$$

$$P_N = (I - K_N H)FP_{N-1}F'(I - K_N H)' + K_N R_N K_N' \,. \tag{8}$$

The residuals are

$$S_N = Z_N - HF\hat{X}_{N-1} = H(X_N - F\hat{X}_{N-1}) + V_N. \tag{9}$$

When the initial values of estimates and covariance matrix ($\hat{X}_0$, P_0) are chosen, Equations (6) - (8) can be used recursively to calculate the estimates.

If the target does not maneuver, the target model describes accurately the target motion, and if the mean of the initial error equals zero, the mean of the residuals should be zero. When the target maneuvers, the target model will become inaccurate, the mean of the residuals will no longer remain zero, and tracking errors occur. The Kalman filter can be viewed as a feedback system. If the gain K_N has appropriate values, the errors can be reduced automatically. But when $GW_N = 0$, K_N and P_N will tend to zero so that the residuals will no longer affect the estimates. (If $GW_N \neq 0$, K_N and P_N eventually will tend to small values, and the residuals cannot significantly affect the estimates.) The tracking errors could become larger and larger, and

eventually the filter will fail in tracking. In other words, the Kalman filter "tracks" the erroneous model rather than the maneuvering target.

From (9), it is seen that the residuals contain two terms. One is measurement noise, with mean equal zero. Another is due to the model error when the target maneuvers. If a simplified smoothing is performed, then the smoothed values of the residuals can represent in some sense the errors produced by target maneuvering. When the smoothed values exceed some threshold values, the adaptive filter will reinitialize the previous value of the covariance P_{N-1}, then the gain K_N can be appropriately adjusted to provide proper feedback and the estimated errors can be reduced automatically. The block diagram of the presented adaptive tracking filter is shown in Figure 1. Here, the portion below the dashed line corresponds to the conventional Kalman-Bucy filter.

The average of the residuals consisting of some previous time points and including the present time point as the simplified smoothed values is chosen, so that

$$\bar{S}_N = [S_N + S_{N-1} + \dots + S_{N-M+1}] / M. \tag{10}$$

Now, the gain formula is given by

$$K_N = FPF'H'[HFPF'H' + R_N]^{-1}, \tag{11}$$

$$P = \begin{cases} P_0 & \text{when } \bar{S}_N > |S_L|, \\ P_{N-1} & \text{when } \bar{S}_N \leq |S_L|, \end{cases} \tag{12}$$

$$P_N = (I - K_N H)FPF'(I - K_N H)' + K_N R_N K_N' . \tag{13}$$

When the values S_L and P_0 are chosen appropriately the estimate errors will be kept within a reasonable range.

This method can work for a variety of target maneuvers, even when the maneuver is small and lasts a long time. The weakness of this method might be that when the target does not maneuver, the rms values of estimate errors are a little larger than for the ordinary Kalman-Bucy filter, but they still will be less than measurement errors. This is because the two factors in residuals S_N mix together, and using the simplified smoothing, the stochastic components cannot be eliminated. So sometimes switching occurs even when the target does not maneuver. This is the cost for simplicity and for keeping small tracking errors when the target maneuvers.

III. THE ADAPTIVE TRACKING FILTER WHEN MEASUREMENTS ARE NONLINEAR

We can generalize this method to nonlinear measurements. When the measurements are made using polar coordinates

$$Z_N = \begin{bmatrix} b_N \\ r_N \end{bmatrix} = BFh(X_N), \tag{14}$$

where b is bearing, r is range,

$$b_N = \tan^{-1} \frac{y_N}{x_N}, \tag{15}$$

$$r_N = \left(x_N^2 + y_N^2\right)^{1/2}. \tag{16}$$

The extended Kalman filter (EKF), and other nonlinear filters, can be adapted to this scheme. For the EKF, the measurement matrix H_N is no longer constant. With the target model in Cartesian Coordinates, it is a function of estimates,

$$H_N = \begin{bmatrix} h_{11N} & 0 & h_{13N} & 0 \\ h_{21N} & 0 & h_{23N} & 0 \end{bmatrix} = \frac{\partial BFh}{\partial X}\bigg|_{X = F\hat{X}_{N-1}}, \tag{17}$$

$$h_{11N} = \left. \frac{y}{x^2 + y^2} \right|_{X = FX_{N-1}} , \quad (18)$$

$$h_{13N} = \left. \frac{-x}{x^2 + y^2} \right|_{X = FX_{N-1}} , \quad (19)$$

$$h_{21N} = \left. \frac{x}{(x^2 + y^2)^{1/2}} \right|_{X = FX_{N-1}} , \quad (20)$$

$$h_{23N} = \left. \frac{y}{(x^2 + y^2)^{1/2}} \right|_{X = FX_{N-1}} . \quad (21)$$

Similar to Equations (9), (11), and (13), the other formulae are

$$S_N = Z_N - BFh(FX_{N-1}) , \quad (22)$$

$$K_N = FPF'H_N'[H_NFPF'H_N' + R_N]^{-1} , \quad (23)$$

$$P_N = (I - K_NH_N)FPF'(I - K_NH_N)' + K_NR_NK_N' . \quad (24)$$

In this case, the estimates x, y can be used to estimate bearing and range as well as the rms errors of bearings and range.

If the measurements are made using two bearings from two sensors in separate locations, this method will result in larger tracking errors. The problem does not arise from the filter scheme itself, but from the sensitivity of the measurement method.

From the geometry, exhibited by Figure 2, it can be seen that, even when the target does not move, with the two-bearing measurements, the range error may be very large even if measurement errors are small. The geometrical relationships are [11]

$$b_1 = \tan^{-1} \frac{r \sin b}{r \cos b + d} , \quad (25)$$

$$b_2 = \tan^{-1} \frac{r \sin b}{r \cos b - d} , \tag{26}$$

$$\tilde{b}_1 = b_1 + \Delta b_1, \tag{27}$$

$$\tilde{b}_2 = b_2 + \Delta b_2, \tag{28}$$

$$\hat{r} = d \left[\frac{1 + \cos^2(\tilde{b}_2 - \tilde{b}_1) - 2\cos(\tilde{b}_2 - \tilde{b}_1)\cos(\tilde{b}_2 + \tilde{b}_1)}{\sin^2(\tilde{b}_2 - \tilde{b}_1)} \right]^{1/2} , \tag{29}$$

$$\hat{b} = \tan^{-1} \left[\frac{\cos(\tilde{b}_2 - \tilde{b}_1) - \cos(\tilde{b}_2 + \tilde{b}_1)}{\sin(\tilde{b}_2 + \tilde{b}_1)} \right] , \tag{30}$$

$$\Delta b = b - \hat{b}, \tag{31}$$

$$\Delta r = r - \hat{r}. \tag{32}$$

Here Δb_1, Δb_2, $\tilde{b}_1$, $\tilde{b}_2$ are measurement errors and measurement bearings respectively; b, r, $\hat{b}$, $\hat{r}$ are real and estimated values of bearing and range respectively. For a numerical example, if

$x = r \cos b = 2000$ yds , $y = r \sin b = 4000$ yds , $d = 200$ yds,

then $r = 4472.14$ yds , $b = 1.1071$ rad. (63.43°)

$b_1 = 1.0680$ rad (61.19°) , $b_2 = 1.1479$ rad. (65.77°).

and the errors shown in Table I result.

Table I. Position Error Sensitivity Relative to Two-Sensor Locations

Δb_1 rad.	+0.05	+0.05	-0.05	-0.05
Δb_2 rad.	+0.05	-0.05	-0.05	+0.05
$\hat{b}$ rad.	1.1572	1.1078	1.0542	1.1039
Δb rad.	-0.0501	-0.0007	0.0529	0.0032
$\hat{r}$ yds.	4583.98	11887.93	4360.68	1984.56
Δr yds.	-111.84	-7415.79	111.46	2487.58

VI. SIMULATION RESULTS

In applying the simplified adaptive tracking filter, two factors need to be considered. One is the number of residuals M used to get the simplified, smoothed values. It is relevant to the accuracy of the smoothed residuals. If the target does not maneuver and the measurement noise is white, then in (9), the mean of S_N equals zero and the standard deviation of the components of the S_N are inversely proportional to the square root of M. However, a large M will reduce the "weight" of the recent measurements. So generally the appropriate value of M is 3-5. In this simulation M = 3.

The second factor is how to choose the threshold level S_L. If the threshold level is low, tracking errors when the target maneuvers can be quite small. However, at a low setting, tracking errors when the target does not maneuver will tend to be large. This is due to occasional unexpected reinitializing. In practice of course, the filter would need to ignore certain extraneous "outlyers."

For a fast-turn maneuver (see Figure 3(a)) different rms errors for different S_L are shown in Figure 4 for linear measurements and in Figure 5 for nonlinear measurements. The standard deviation of the measurement noise is 100 yds. (in x, y) for the linear observation and 100 yds. in range and 0.035 rad. in bearing for the nonlinear observation. The results are about as expected. One and three tenths (1.3) times the standard deviation of measurement noise is selected for S_L as a compromise between transient error (due to maneuver) and steady error. Of course, more analysis is necessary to "optimize" this selection.

As a comparison with the method of Chan et al [6], [7] almost the same target maneuvering and data is used. There is just a little difference. When the target maneuvers it is assumed that the magnitude of the target velocity is

constant and the direction of the target velocity changes uniformly with time rather than discontinuously. The equations of the adaptive tracking filter are as given in Section II with data $T = 10s$, $S_L = 130$ yds and

$$\Gamma_N = 0. \quad R_N = \begin{bmatrix} 10000 & 500 \\ 500 & 10000 \end{bmatrix}, \; N = 1, 2, 3, \ldots, 100.$$

The initial conditions of the target are given by $x_0 = 1000$ yds, $x_0 = 0$, $y_0 = 10000$ yds, $y_0 = -15$ yd/s. The mean of the initial errors is zero, and

$$P_0 = \begin{bmatrix} 10,000 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 10,000 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}.$$

For the first target motion (see Figure 3,a), the target moves in a straight line at constant speed until $t = 400s$ ($N = 40$), it then makes a fast turn moving with angular velocity $\dot{\theta} = -\pi/100$ rad/s until after 50s it has executed a 90° turn. It then continues in a straight line at constant speed on this new course. The second motion (Figure 3,b) is much like the first, however the turn is slower with $\dot{\theta} = -\pi/400$ rad/s. In this case the turn takes 200s. In the third motion (Figure 3,c), the target carries out a turn identical to that in the second motion, then assumes a course parallel to its original one by performing a fast turn with $\dot{\theta} = \pi/100$ rad/s. In the fourth motion (Figure 3,d), the target performs a fast circular turn ($\dot{\theta} = -\pi/100$ rad/s). Completing it in 200s, it then continues on this original course. The last motion (Figure 3,d) is a very slow 90° turn with $\dot{\theta} = -\pi/1000$ rad/s, which takes 500 sec to complete. In all of these motions, the maneuver begins at $t = 400s$ ($N = 40$).

For each kind of target motion, 50 samples are run and the rms errors are computed. The standard deviation of the measurement noise is as given above

with sample rms values about ±20% about this value. Figure 6 shows the rms tracking errors for the corresponding motions with linear observations.

For nonlinear measurements,

$$R_N = \begin{bmatrix} 0.0012 & 0.175 \\ 0.175 & 10000 \end{bmatrix}, \qquad N = 1, 2, \ldots, 1000,$$

$S_{L1} = 0.045$ rad and $S_{L2} = 130$ yds.

Again, the standard deviation of the measurement noise is 100 yds. in range and 0.035 rad. in bearing with the sample rms values approximately ±20% about this sample mean. The rms tracking errors of bearing and range for the trajectories given in Figure 3 for the above are plotted in Figure 7.

According to these results, the adaptive tracking filter ATF is very effective, and the errors are significantly less than by the method of Chan, et al [6], [7]. A rough comparison with this popular method is made in Table II.

Table II. Comparison of Methods

Max. RMS Position Error, Yds.			
Motion	ATF	[6]	[7]
Figure 1(a)	91-121	180	260-350
Figure 1(b)	82-88	190	290
Figure 1(c)	91-121	215	300
Figure 1(d)	121-122	*	360
Figure 1(e)	89-90	*	*

*Not available

Maximum velocity errors by the proposed method are approximately 10 to 50 percent less than the method of Chan, et al using their method as the base for comparison.

V. CONCLUSION

A simple adaptive tracking filter has been presented here. For each measurement sample, the filter produces the residual sample and does very simple smoothing. In turn, the smoothed residuals adapt the filter gain. This method can be used for either linear measurements or nonlinear measurements. The resulting tracking errors are small for all cases studied. Also, the method is very simple, robust and involves few computations. No prior knowledge of the maneuvering characteristics of the target is required. If it is known how frequently the target maneuvers, the threshold level can be adjusted to optimize the method's performance. If there is an operator monitoring the smoothed residuals, the threshold level can easily be adjusted to achieve better tracking performance.

Since a class of "bilinear" filters has shown improved performance over extended Kalman filters for typical nonmaneuvering cases and the above nonlinear observations [8] and [9], it is expected that the planned adaptation of the present algorithm to the "bilinear" filter will yield even better performance than that shown above.

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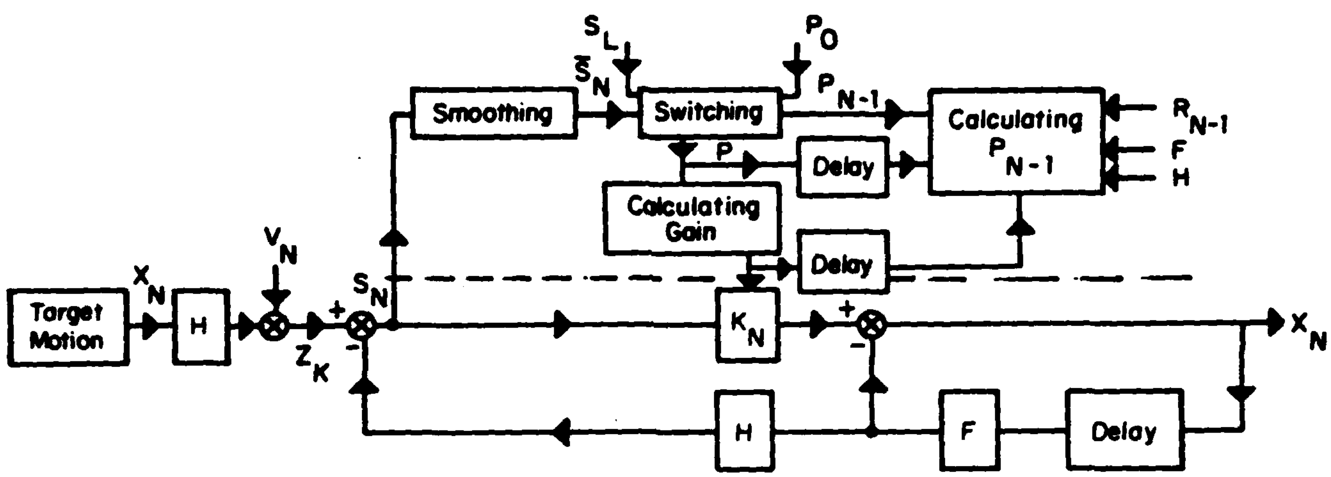

Figure 1. Adaptive Tracking Filter

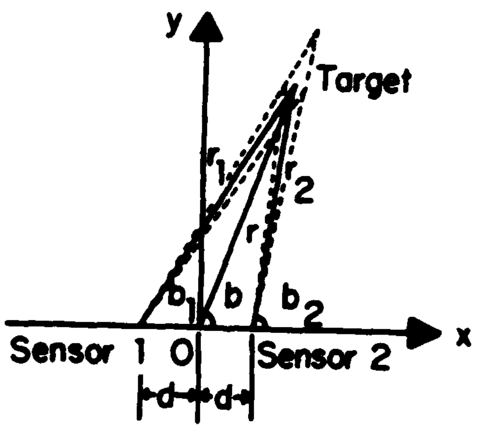

Figure 2: Geometry for Two-Bearing Measurements.

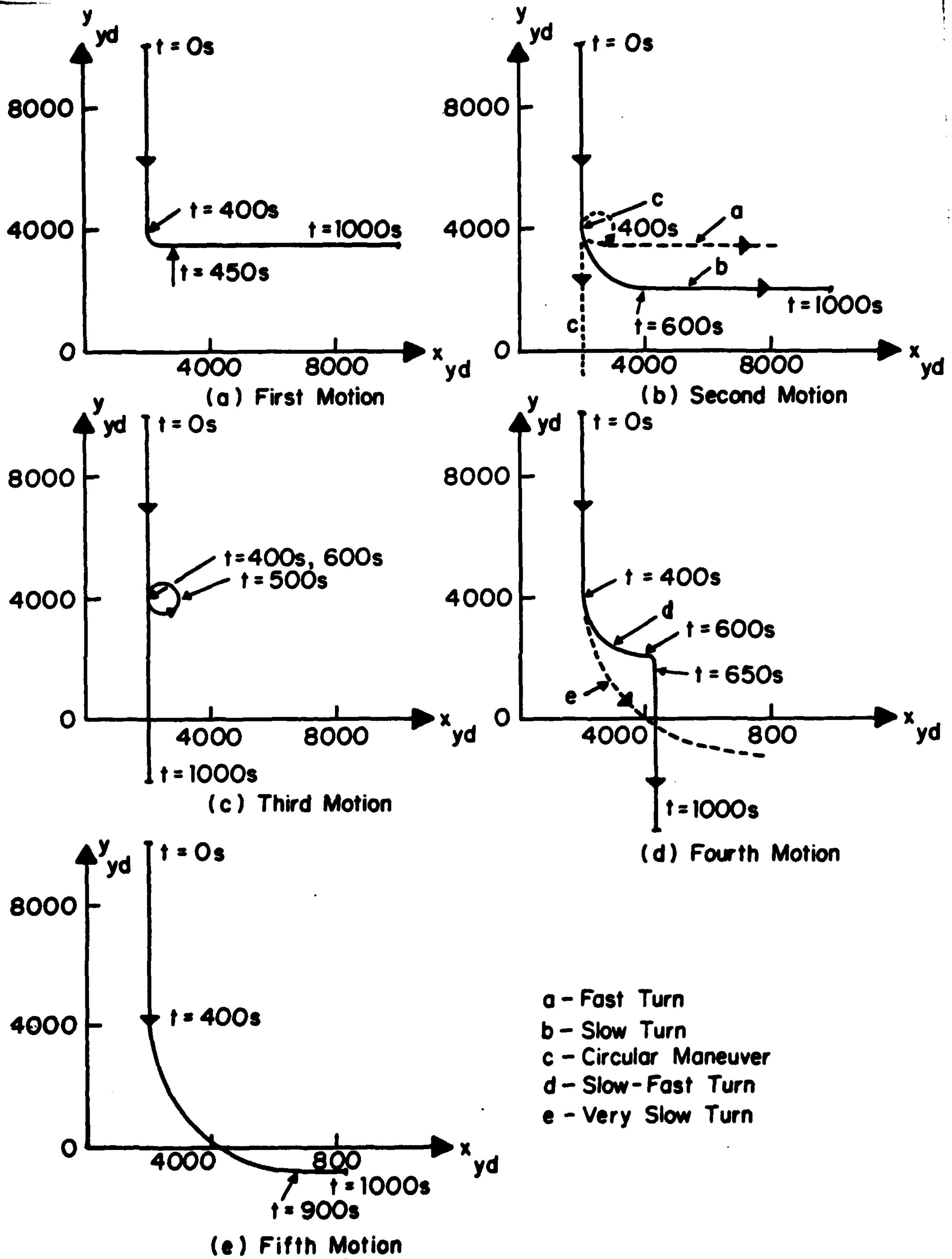

Figure 3. Simulated Target Trajectories.

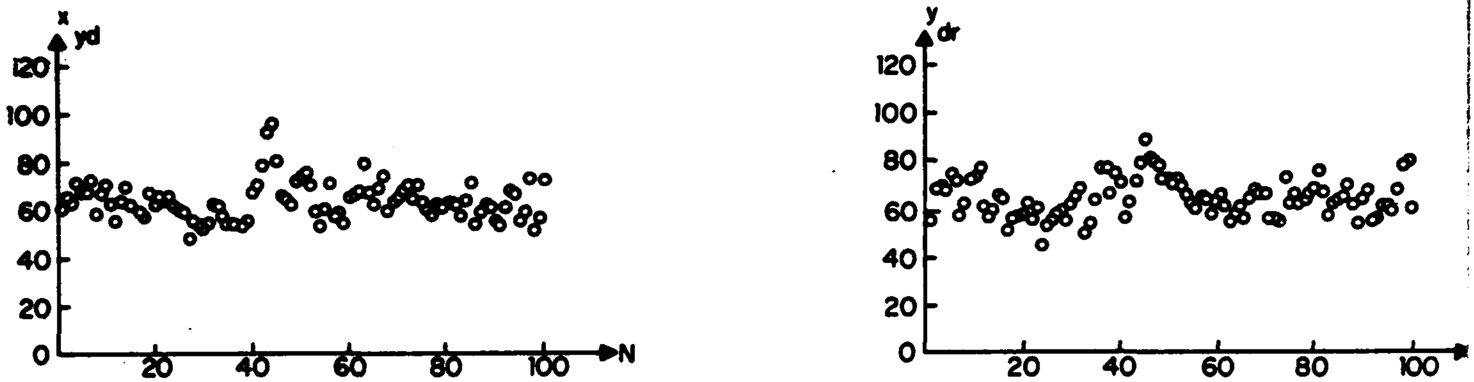

Fig. 4a rms tracking errors in position, fast turn. S_L = 100 yd

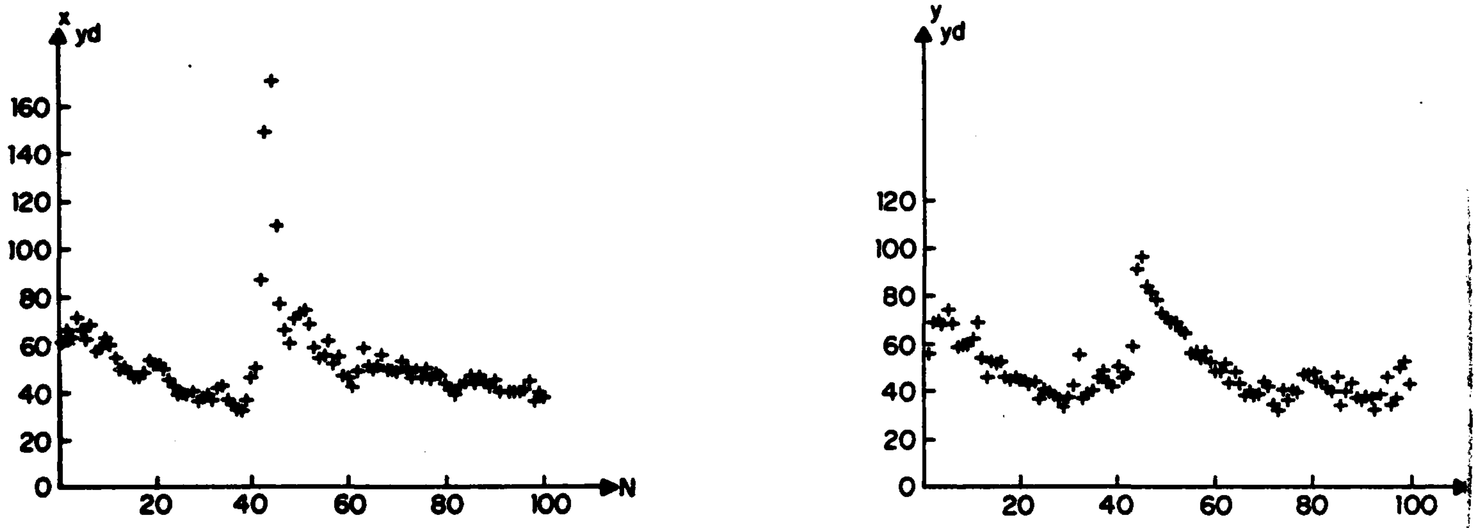

Fig. 4b rms tracking errors in position, fast turn. S_L = 160 yd

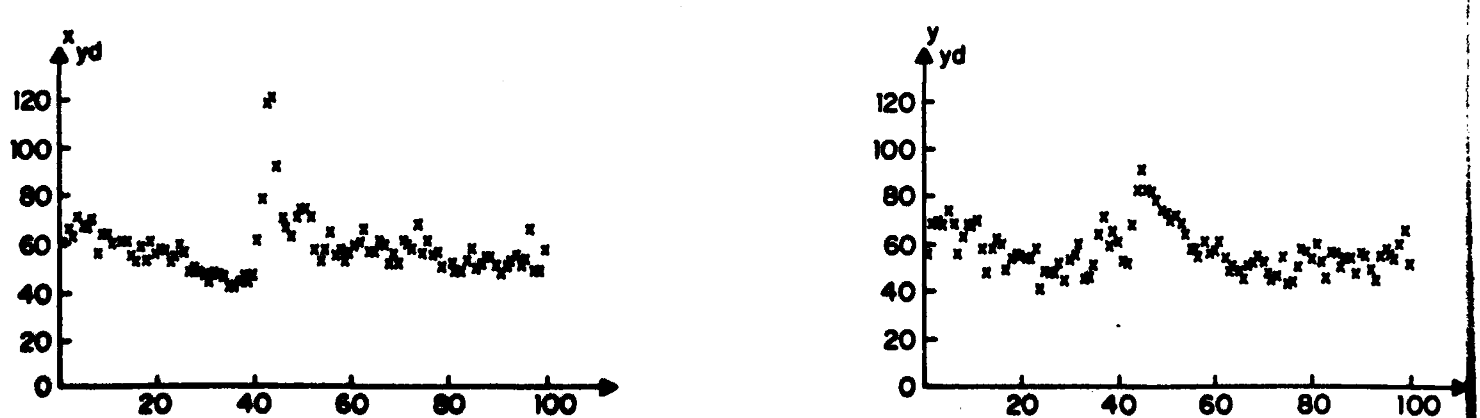

Fig. 4c rms tracking errors in position, fast turn. S_L = 130 yd

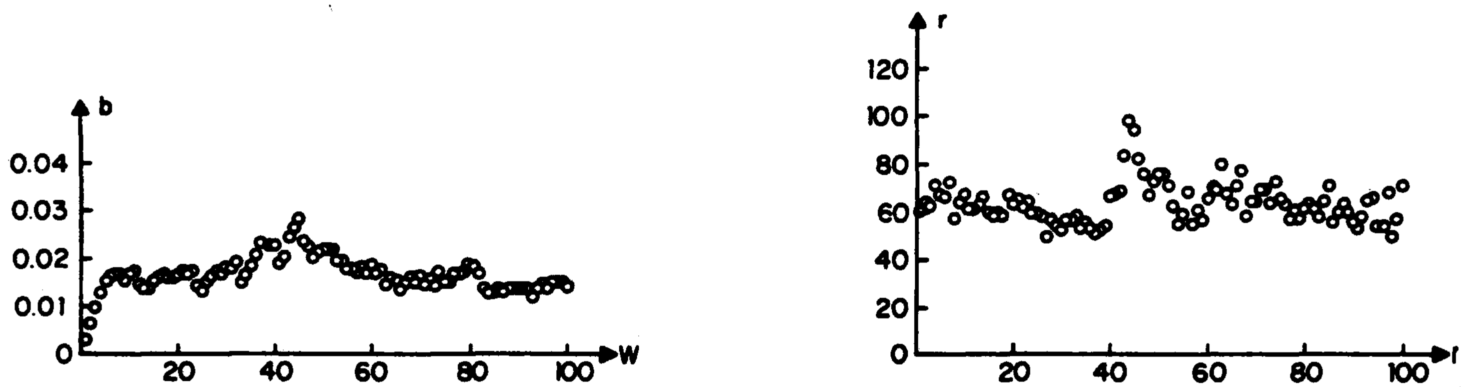

Fig. 5a rms tracking errors in position, fast turn, $S_{L_1} = 0.035_{rad}$, $S_{L_2} = 100$ yd

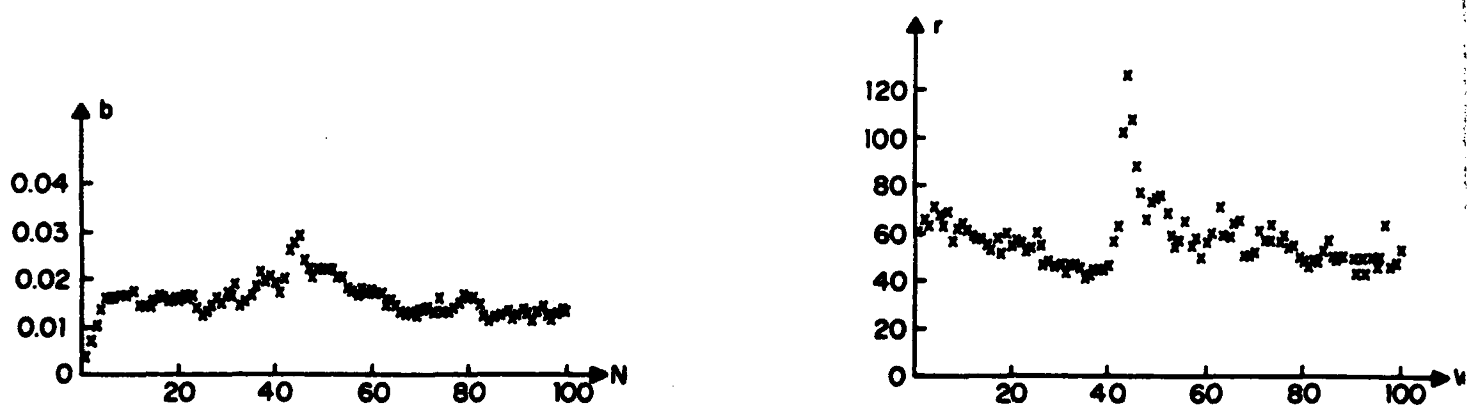

Fig. 5b rms tracking errors in position, fast turn, $S_{L_1} = 0.045_{rad}$, $S_{L_2} = 130$ yd

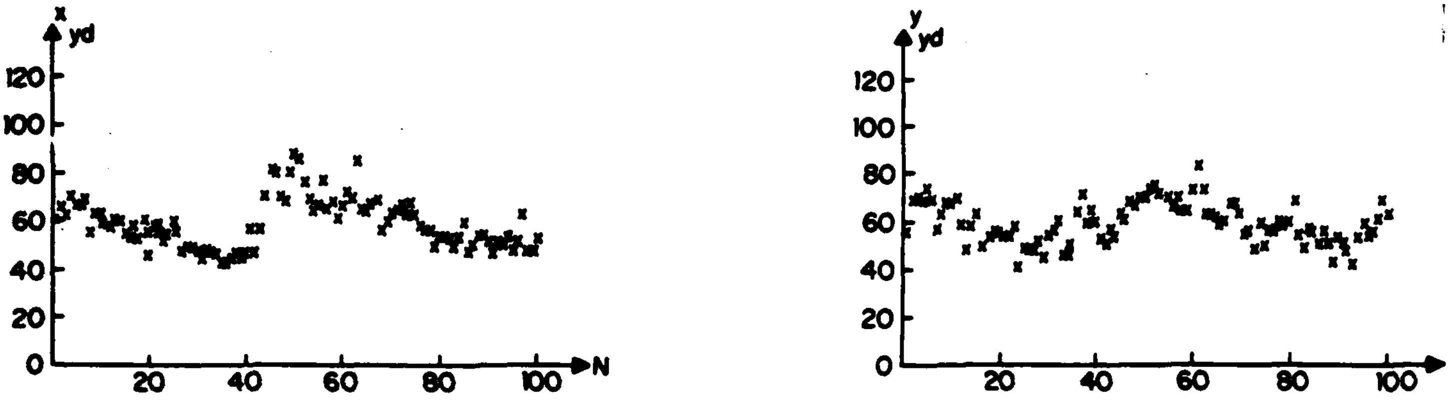

Fig. 6a rms tracking errors in position, slow turn, S_L = 130 yd

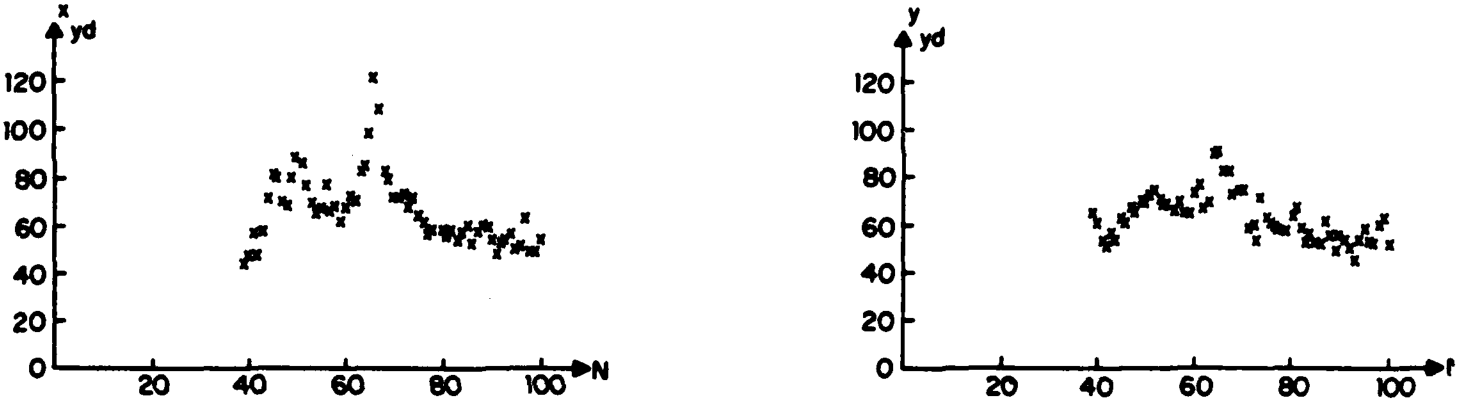

Fig. 6b rms tracking errors in position, slow-fast turn, S_L = 130 yd

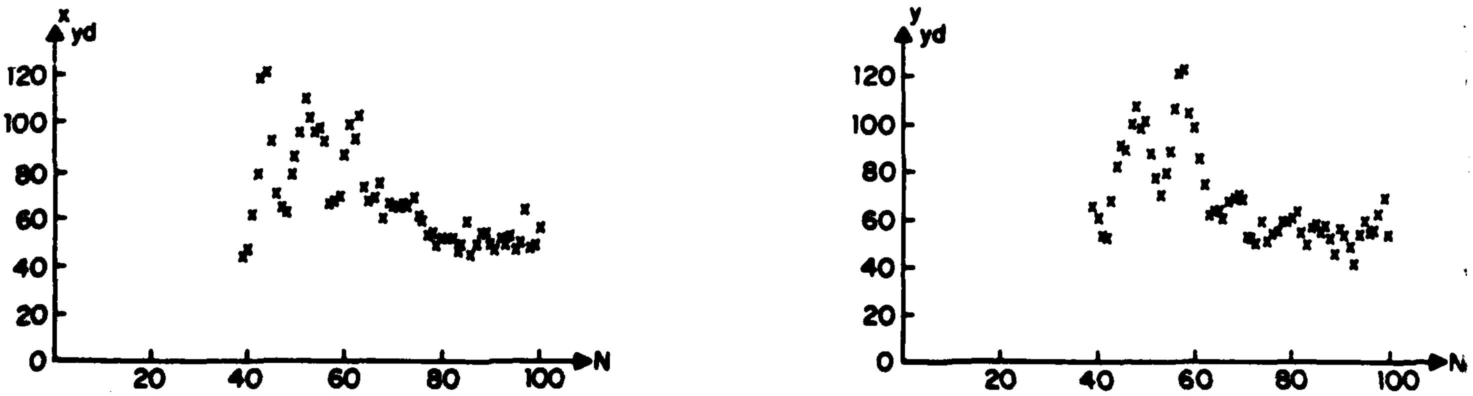

Fig. 6c rms tracking errors in position, circle turn, S_l = 130 yd

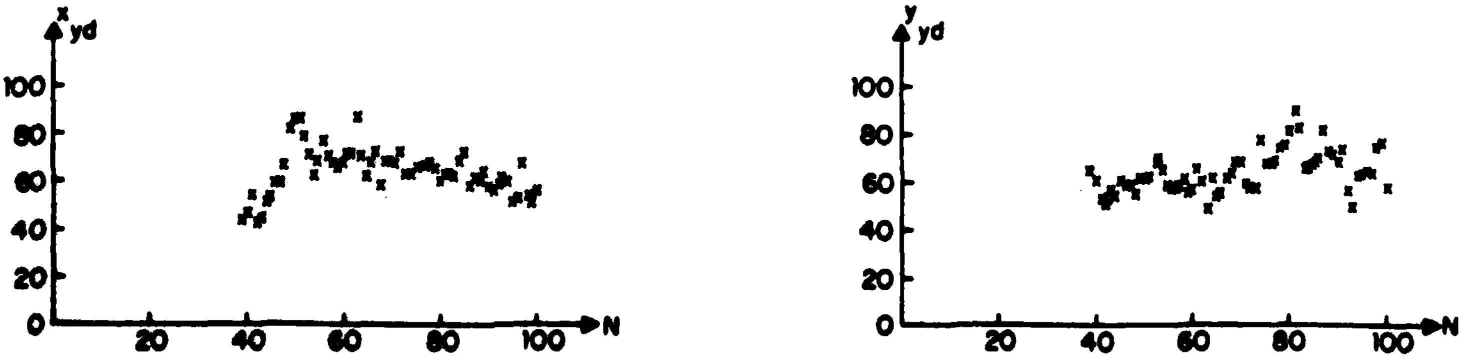

Fig. 6d rms tracking errors in position, very slow turn, S_L = 130 yd

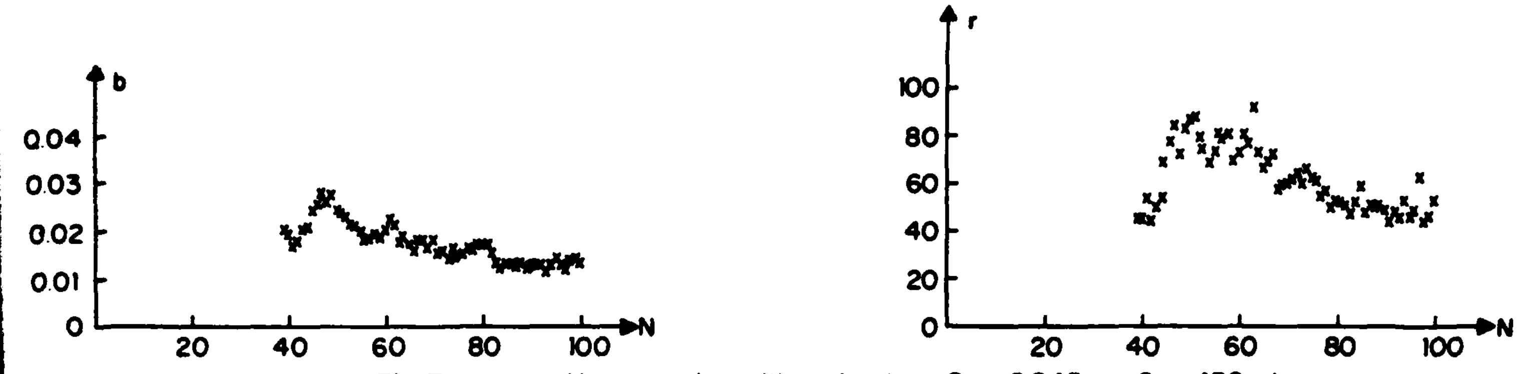

Fig. 7a rms tracking errors in position, slow turn, $S_{L_1} = 0.045_{rad}$, $S_{L_2} = 130$ yd

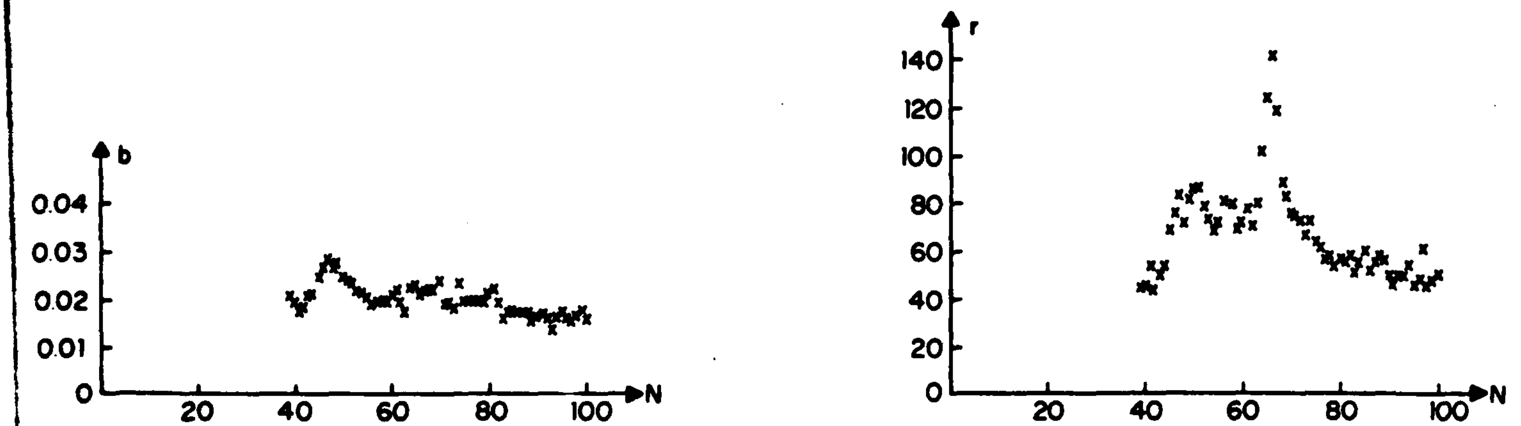

Fig. 7b rms tracking errors in position, slow turn, $S_{L_1} = 0.045_{rad}$, $S_{L_2} = 130$ yd

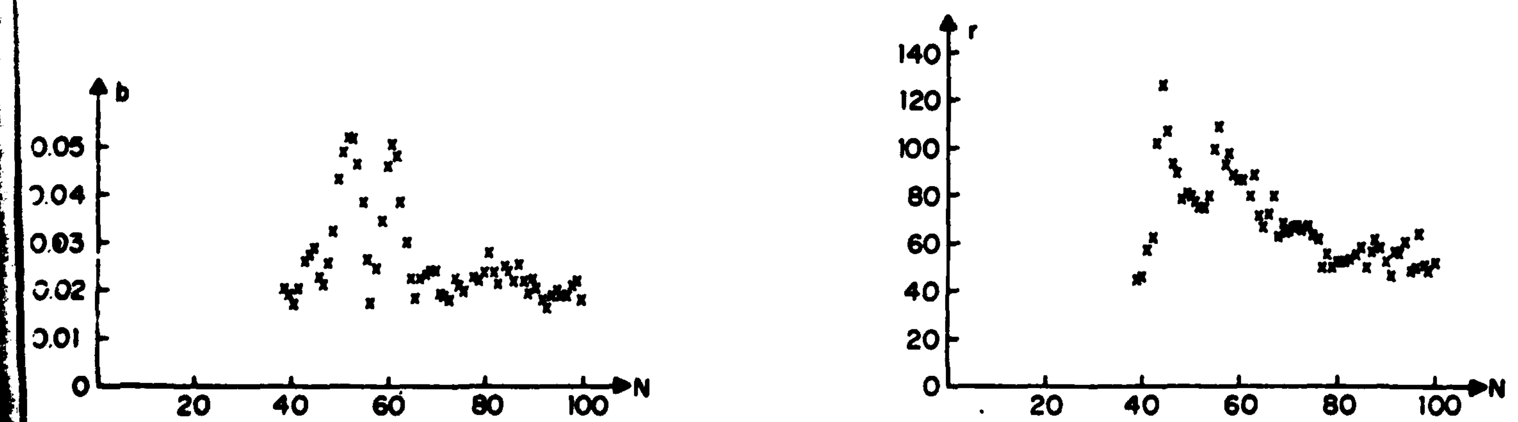

Fig. 7c rms tracking errors in position, circle turn, $S_{L_1} = 0.045_{rad}$, $S_{L_2} = 130$ yd